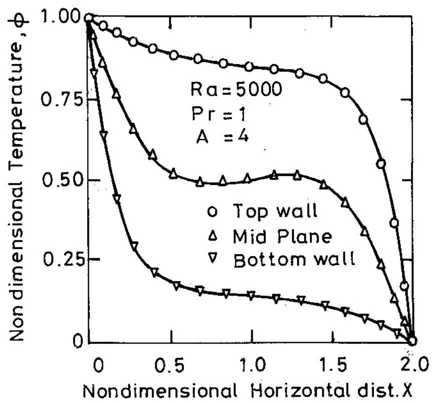

Fig. 4 Temperature profiles across the cavity due to convection.

The difference in $\overline{Nu}_R$ obtained from Eqs. (8) and (9) can be as high as 15%. Since the present correlation covers a wide range of aspect ratios and is mathematically more rigorous, it is recommended as a replacement for the parallel plate formula in the range of parameters shown in Table 1. The present calculations are for the laminar flow regime. At high aspect ratios when the Ra_H exceeds 10^9 or so, the flow may change to the turbulent regime. Hence, correlations for turbulent flow in cavities have to be used in determining the free convection heat transfer. Regarding the radiation, even at high aspect ratios, the present calculations are superior to the parallel plate formula. Therefore, the correlation for the radiation heat transfer can be used to predict the radiant heat flux, but with a caution that a laminar solution is being extrapolated to the turbulent regime as regards the wall temperature distribution, and to that extent the present correlation may be in error. However, at low aspect ratios, but high Ra due to large temperature differences, the temperature profiles on the top and bottom have to be obtained by solving the flow equations which take into account the effect of turbulence. Also at this point it needs to be emphasised that the above calculations are not valid in situations where the fluid flow is three-dimensional. Only if the depth of the cavity is very large compared to the spacing, two-dimensional approximation is reasonable.

Conclusions

Detailed numerical calculations have been done in cavities with A in the range of 2–20 when both free convection and surface radiation are present. The use of temperature profiles obtained by solving the flow equations for determining the heat flux, along with a detailed enclosure analysis, clearly expose the hollowness of the simple parallel plate formula for radiation, which neither considers this effect nor the effect of aspect ratio. At the same time, for cavities with $A \geq 2$, the decoupling of the two mechanisms of heat transfer is possible, and on this basis comprehensive correlations for radiation Nusselt number and convection Nusselt number have been proposed.

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Natural Convection in Horizontal Porous Annuli with Circumferential Baffles

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Introduction

To suppress natural convection within insulation is one of the important considerations in process design. Recently, Bejan[1,2] has shown that using internal partitions can effectively reduce convective heat losses. For pipes, this can be implemented by insertion of radial baffles in the insulation.[3] In this Note, an alternative approach, i.e., using circumferential baffles, is investigated. The same technique has been applied to air-filled annuli by Yang et al.[4] and proved to be successful. In this study, designs that use only one single baffle (cases I and II) are examined first, followed by the design that requires two baffles (case III). For the case of single baffle, two baffle locations are considered: one is directly above the inner cylinder (case I), while the other is underneath it (case II). Numerical results are obtained for a complete set of baffle angular span (0 deg $\leq \delta \leq$ 360 deg) and a range of Rayleigh number of practical interest ($50 \leq Ra \leq 500$).

Formulation and Numerical Method

For most pipe insulation available today, they can be adequately modeled as a porous annulus. For a typical application, the inner surface is heated at a constant temperature T_i, while the outer surface is maintained at the ambient temperature T_0 ($T_i > T_0$). Having invoked the Boussinesq ap-

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proximation, the dimensionless governing equations based on Darcy's law are given by[3]

$$\frac{\partial V_\theta}{\partial R} + \frac{V_\theta}{R} - \frac{1}{R}\frac{\partial V_r}{\partial \theta} = Ra\left(\cos\theta\,\frac{\partial \Theta}{\partial R} - \frac{\sin\theta}{R}\frac{\partial \Theta}{\partial \theta}\right) \tag{1}$$

$$V_r\frac{\partial \Theta}{\partial R} + \frac{V_\theta}{R}\frac{\partial \Theta}{\partial \theta} = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial \Theta}{\partial R}\right) + \frac{1}{R^2}\frac{\partial^2 \Theta}{\partial \theta^2} \tag{2}$$

with the boundary conditions given by

$$\Theta = \tfrac{1}{2}, \quad V_r = 0, \qquad \text{on the inner wall} \tag{3a}$$

$$\Theta = -\tfrac{1}{2}, \quad V_r = 0, \qquad \text{on the outer wall} \tag{3b}$$

In the above expressions, Ra is the modified Rayleigh number defined by $Kg\beta(T_i - T_0)D/\alpha\nu$, where K is the permeability of the pipe insulation, β the coefficient of thermal expansion, and D the gap width of the annulus. Also, Θ is the dimensionless temperature given by $(T - T_m)/(T_i - T_0)$, in which T_m is the mean temperature. Although the insertion of baffles may lead to a possible modification of the medium characteristics near the baffle (i.e., the variation of porosity), it has been pointed out[5,6] that the relative increase of heat transfer due to porosity variation near the wall is almost balanced by the reduction of heat transfer due to the imposed no-slip condition on a rigid wall. Therefore, these higher-order effects are not considered here for simplicity and consistency. As for the baffles, they are assumed to be very thin such that the radial temperature gradient is negligible.

To solve the simultaneous equations defined above, the following coordinate transformation is introduced to facilitate calculations. For brevity, the details of the coordinate transformation are omitted here, but can be found in Ref. 7:

$$x = \frac{\ln R - \ln R_i}{\ln R_0 - \ln R_i} - \frac{1}{2}, \qquad y = \frac{\theta - \theta_i}{\theta_0 - \theta_i} \tag{4}$$

Since the geometry is symmetrical to the vertical diameter, only half of the domain is needed for computation. Thus, the governing equations in terms of stream function ψ are transformed to

$$x_0^2\frac{\partial^2\psi}{\partial x^2} + y_0^2\frac{\partial^2\psi}{\partial y^2} = -Ra(R_iR_0)^{1/2}\left(\frac{R_0}{R_i}\right)^x \times \left[x_0\frac{\partial \Theta}{\partial x}\cos\left(\frac{y}{y_0} + \theta_m\right) - y_0\frac{\partial \Theta}{\partial y}\sin\left(\frac{y}{y_0} + \theta_m\right)\right] \tag{5}$$

$$x_0y_0\left(\frac{\partial\psi}{\partial y}\frac{\partial \Theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial \Theta}{\partial y}\right) = x_0^2\frac{\partial^2 \Theta}{\partial x^2} + y_0^2\frac{\partial^2 \Theta}{\partial y^2} \tag{6}$$

where $x_0 = 1/\ln(R_0/R_i)$, $y_0 = 1/\pi$, $\theta_0 = \pi/2$, $\theta_i = -\pi/2$, and $\theta_m = (\theta_0 + \theta_i)/2$.

The transformed governing equations and boundary conditions are solved numerically by employing a standard finite difference method. The details of the numerical scheme are omitted here again for brevity and may be found in Ref. 3. Uniform grids, 51 × 121 in the transformed domain, are used for the present study. To validate the numerical results thus obtained, the solutions have been compared with those reported in the literature for the case of a plain annulus. The agreement is very good as reported in Ref. 3.

Since the emphasis of the present study is placed on the effects of baffle angular extent on the heat transfer results, computations are restricted to the case of $r_i = 1$ and $D = 2$. In addition, the baffle is positioned midway between the cylinders, at $(R_0 + R_i)/2$, throughout the study. For simplicity, it is further assumed that the thermal conductivity of baffles is the same as that of the pipe insulation. This is a reasonable assumption since the baffle used in applications must have a thermal conductivity smaller than that of the pipe insulation to effectively reduce the convective heat loss. Under this assumption, the results thus obtained form an upper bound of the problem.

Results and Discussion

The flow and temperature fields for a pipe insulation with three baffle arrangements are shown in Fig. 1. When the baffle is short (e.g., $\delta = 30$ deg), the flow and temperature fields are almost the same as those of an unobstructed annulus. For the first case, a weak secondary flow appears in the region directly above the baffle as δ increases between 60–90 deg. It disappears when δ is increased further. With an increase in the baffle angular extent, the strength of convective cell is considerably decreased. The thermal plume observed in a plain annulus is unable to develop fully because of the blockage of convective flow. As a result, the temperature gradient on the outer surface is greatly reduced.

The flow and temperature fields for the second case are very different from the first case. For $\delta \le 90$ deg, the streamline and isotherm patterns on the upper half of the annulus closely resemble those for an unobstructed annulus, which indicates that the presence of a circumferential baffle has negligible effects on the heat transfer result. As δ increases, the fluid at the bottom half of the annulus becomes stagnant due to the deflection of flow by the baffle. From the isotherm patterns, it is evident that the heat transfer mode in the lower region has changed from weak convection to primary conduction.

Since the baffle arrangement in the third case is a combination of that in the previous two cases, it is expected that some of the characteristics observed will be retained. This is especially true when the baffles are short (e.g., $\delta \le 60$ deg), the flow and isotherm patterns look as if they were obtained by superposition from the previous cases. Since the flow resistance is doubled, the strength of convective cell is significantly weakened. Heat loss by convection is thus effectively suppressed.

The interest in many applications is the heat transfer coefficient. The heat transfer coefficient in terms of the Nusselt number is given by

$$Nu = \frac{hD}{k} = -\frac{1}{R_i \ln(R_0/R_i)}\int_{-1/2}^{1/2}\left.\frac{\partial \Theta}{\partial x}\right|_{x=1/2} dy \tag{7}$$

The heat transfer results thus obtained are most informative if the Nusselt number is normalized by its conduction value.

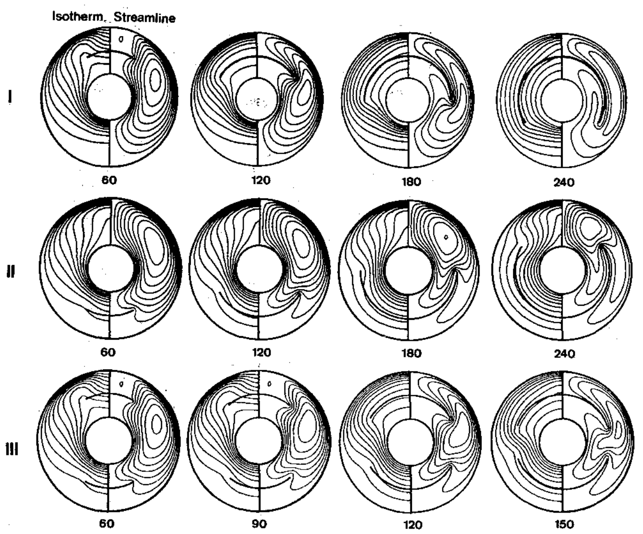

Fig. 1 Flow and temperature fields for an annulus with circumferential baffles ($Ra = 100$, $\Delta\psi = 1$, and $\Delta\Theta = 0.1$).

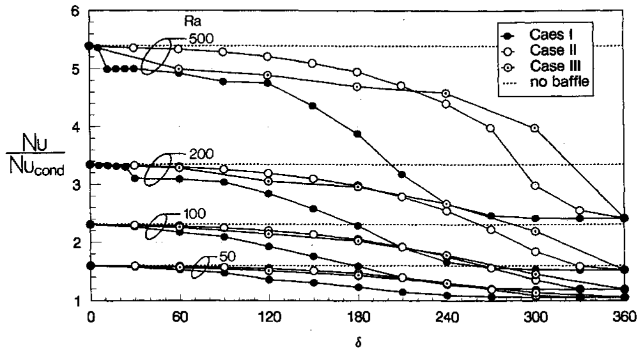

Fig. 2 Effects of baffle angular span on the heat transfer results.

It is easy to show that the conduction Nusselt number can be evaluated as

$$Nu_{\text{cond}} = \frac{1}{R_i \ln(R_0/R_i)} \tag{8}$$

The normalized Nusselt numbers are plotted in Fig. 2, from which it is confirmed that circumferential baffles are effective in reducing heat losses from a pipe insulation. The reduction in heat losses increases with the baffle angular span, and it reaches the maximum when the annulus is completely partitioned into two sections. For $Ra = 50$, the maximum reduction in heat loss is 34%, and it is 48, 54, and 55% for $Ra = 100$, 200, and 500, respectively.

Conclusions

In this study, the feasibility of using circumferential baffles to reduce convective heat losses from pipe insulation has been successfully demonstrated. It shows that the effectiveness of pipe insulation is directly proportional to its baffle angular extent. The maximum reduction in heat loss always occurs at $\delta = 360$ deg, when the pipe insulation is completely partitioned into two sections. This finding has an important implication for engineering practice: it is more effective to apply multiple layers of insulation, with each layer being separated by an impermeable partition, than to use only one single layer of insulation with the same thickness. This suggested practice is simple, but it is often overlooked by engineers.

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Transient Compressible Flow in Variable Permeability Media

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Introduction

In many applications, decomposing polymers are subjected to rapid heating. The high temperatures induce chemical reactions within the polymer which cause the formation and flow of pyrolysis gases. Temperature gradients cause steep property variations, especially for permeability, which may vary by several orders of magnitude across the narrow pyrolysis zone. Accurate estimates of the pore pressure is required for mechanical stress and failure analysis of the polymer.

Pressure is predicted with the pore pressure equation, which is a combination of the conservation of mass and Darcy's law for porous media. For transient one-dimensional compressible flow, the pore pressure equation is

$$\frac{\partial}{\partial x}\left(\rho \frac{K}{\mu}\frac{\partial P}{\partial x}\right) = \frac{\partial}{\partial t}(\phi\rho) \tag{1}$$

where P is pressure, ρ is fluid density, μ is fluid viscosity, K is permeability, and ϕ is porosity. For highly compressible flow, both Muskat[1] and Morrison[2] have presented exact solutions to the pore pressure equation. Neither successful measurement of pore pressure in a decomposing composite, nor exact transient solutions to Eq. (1) for variable permeability compressible flow, were found in the literature.

Model

An isothermal variable permeability exact solution, suitable for verification purposes, will be developed for the pore pressure equation. The schematic of the problem, and boundary and initial conditions, are shown in Fig. 1a. In the pyrolysis zone the permeability varies approximately exponentially with distance. However, the exponential function is not very suitable for an exact solution; thus another, somewhat more severe assumption, is that the permeability varies nearly inversely with distance. For a given permeability change, Fig. 1b compares the exponential function to the assumed permeability variation, shown as follows:

$$K = \frac{K_L}{x/L + K_L/K_s} \qquad 0 < K_L/K_s << 1 \tag{2}$$

where L is the distance from the surface to the point where $K = K_L$, which is considered the edge of the domain for computational purposes. Equation (2) and the ideal gas law can be substituted into Eq. (1) to obtain

$$\frac{\partial}{\partial x}\left(\frac{K_L}{x/L + K_L/K_s}\frac{PM}{\mu RT}\frac{\partial P}{\partial x}\right) = \frac{\partial}{\partial t}\left(\phi\frac{PM}{RT}\right) \tag{3}$$

For this study, the porosity, viscosity, temperature T, and molecular weight M, are assumed constant. For a step bound-

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